



**INDIAN SCHOOL MUSCAT
HALF YEARLY EXAMINATION 2022
MATHEMATICS (041)**



CLASS : XII
DATE: 15.09.22

TIME ALLOTTED : 3 HRS.
MAXIMUM MARKS: 80

GENERAL INSTRUCTIONS:

- (a) This question paper consists of 38 questions divided into five sections A, B, C, D and E.
 - (b) Section-A comprises of 20 questions of **one mark** each (Q1 to Q 20).
 - (c) Section-B comprises of 5 questions of **two marks** each (Q 21 to Q 25).
 - (d) Section-C comprises of 8 questions of **three marks** each (Q 26 to Q 33).
 - (e) Section-D comprises of 2 Case-study questions with **4 marks** each (Q 34 and Q 35).
 - (f) Section- E comprises of 3 questions of **6 marks** each (Q 36 to Q 38).
- There is no overall choice. However, internal choice has been provided.

SECTION – A (Questions 1 to 20 carry 1 mark each)

(i) MULTIPLE CHOICE QUESTIONS:

1. The point on the curve $y^2 = 8x$ for which the abscissa and ordinate change at the same rate is:
(a) (1, 8) (b) (0, 0) (c) (2, 4) (d) (3, -2)
2. $\int \frac{\sin\sqrt{x}}{\sqrt{x}} dx$ equals
(a) $-2\cos\sqrt{x} + C$ (b) $\frac{1}{2} \cos\sqrt{x} + C$ (c) $2\sin\sqrt{x} + C$ (d) $\log|\sqrt{x}| + C$
3. The value of k so that the function $f(x) = \begin{cases} kx & \text{if } x < 0 \\ |x| & \text{if } x \geq 0 \end{cases}$ is continuous at $x = 0$ is
(a) 3 (b) -3 (c) 0 (d) does not exist
4. If for any 2×2 square matrix A , if $A(\text{adj } A) = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$, then the value of $|A|$ is
(a) 64 (b) 1 (c) 8 (d) 0
5. If $f(x) = 9^{x^2+2x}$, then $f'(-1)$ is
(a) 0 (b) 2 (c) 4 (d) -2
6. $\int \sqrt{1 + \sin 2x} dx =$
(a) $\tan x - \cot x + C$ (b) $\sin x - \cos x + C$
(c) $\sin x + \cos x + C$ (d) $\cos x - \sin x + C$
7. If A is a square matrix such that $A^2 = A$, then $(I + A)^2 - 3A$ is

- (a) I (b) $2A$ (c) $3I$ (d) A

8. If $y = a \sin^3 t$, $x = a \cos^3 t$, then $\frac{dy}{dx}$ at $t = \frac{3\pi}{4}$ is

- (a) -1 (b) $-\frac{1}{\sqrt{3}}$ (c) $-\sqrt{3}$ (d) 1

9. If the rate of change of volume of a sphere is equal to the rate of change of its radius, then its radius is

- (a) $\frac{1}{2\pi}$ units (b) $\frac{1}{4\pi}$ units (c) $\frac{1}{2\sqrt{\pi}}$ units (d) $\frac{1}{\sqrt{\pi}}$ units

10. $\int x^2 \sin x^3 dx = ?$

- (a) $-\frac{1}{3} \cos x^3 + C$ (b) $\frac{1}{3} \cos x^3 + C$ (c) $\frac{1}{3} \sin x^3 + C$ (d) $-\frac{1}{3} \sin x^3 + C$

(ii) **FILL IN THE BLANKS:**

11. Assume that the function $f(x)$ has its second derivative at $x = c$ such that $f'(c) = 0$ and $f''(c) > 0$, then $x = c$ is a point of -----.

12. $\int e^{3 \log x} \cdot x^4 dx = \underline{\hspace{2cm}}$

13. If $y = \cos\left(\frac{\pi}{2} - \sin^{-1} x\right)$, where $x \in [-1, 1]$, then $\frac{dy}{dx} = \underline{\hspace{2cm}}$.

14. If A and B are symmetric matrices, then $(AB - BA)$ is a ----- matrix.

(iii) **TRUE OR FALSE :**

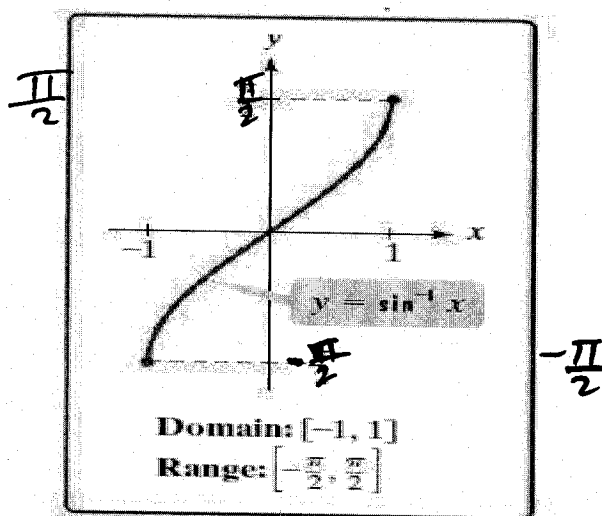
15. The point at which the function $f(x) = |2x + 3|$ is not differentiable is $\frac{3}{2}$.

16. The derivative of a function is unique but a function can have infinite anti-derivatives.

17. The cofactor of the element a_{21} of the determinant $\begin{vmatrix} 5 & 6 & -3 \\ -4 & 3 & 2 \\ -4 & -7 & 3 \end{vmatrix}$ is -3 .

18. The total revenue in ₹ received from the sale of x units of an article is given by $R(x) = 3x^2 + 36x + 5$. The marginal revenue (in ₹) when $x = 15$ is ₹ 126.

(iv) **ANSWER Q.19 & Q.20 BASED ON THE GIVEN GRAPH (SOURCE-BASED QUESTION):**



19. Write one branch of $\sin^{-1} x$ other than the principal branch.

20. What is the value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$?

SECTION – B (Questions 21 to 25 carry 2 mark each)

21. Check if the relation R on the set $A = \{1, 2, 3, 4, 5, 6\}$ defined as $R = \{(x, y): y \text{ is divisible by } x\}$ is (i) symmetric (ii) transitive.
(OR)
Show that the relation R in the set of real numbers, defined as $R = \{(a, b): a \leq b^2\}$ is neither reflexive nor symmetric nor transitive.
22. Differentiate: $\tan^{-1} \left(\frac{1+\cos x}{\sin x} \right)$ with respect to x .
(OR)
Find the derivative of $x^{\log x}$ with respect to $\log x$.
23. Find $\int \frac{1}{(1-x)(2-x)} dx$
(OR)
Find $\int \frac{(2x-5)e^{2x}}{(2x-3)^3} dx$
24. Show that the function f in $A = R - \left\{ \frac{2}{3} \right\}$ defined as $f(x) = \frac{4x+3}{6x-4}$ is one-one.
25. Evaluate : $\int \sec^4 x \tan x \, dx$

SECTION – C (Questions 26 to 33 carry 3 mark each)

26. If $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find x and y so that $A^2 - xI - yA = O$. Hence find A^{-1} .
(OR)
Find the value of x if $\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = O$
27. Evaluate : $\int x^2 \tan^{-1} x \, dx$.
28. The volume of a cube is increasing at the rate of $8 \text{ cm}^3/\text{s}$. At what rate is its surface area increasing when the length of its edge is 12 cm ?
29. Find the equation of the line joining $A(1, 3)$ and $B(0, 0)$ using determinants and find k if $D(k, 0)$ is a point such that area of $\triangle ABD$ is 3 sq. units .
30. Find the value of : $\tan^{-1} \left[2 \sin \left(2 \cos^{-1} \left(-\frac{\sqrt{3}}{2} \right) \right) + 2\sqrt{3} \right]$
(OR)
Find the principal value of : $\tan^{-1} \left(\tan \frac{5\pi}{6} \right) + \cos^{-1} \left(\cos \frac{13\pi}{3} \right) - \cot^{-1} (-\sqrt{3})$
31. If $y = (\sin^{-1} x)^2$, prove that $(1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - 2 = 0$
32. Find the absolute maximum and absolute minimum values of the function given by $f(x) = \sin^2 x - \cos x$, $x \in [0, \pi]$
(OR)
Find the intervals in which the function f given by $f(x) = x^4 - 2x^2$ is
(i) strictly increasing (ii) strictly decreasing.
33. Evaluate : $\int \frac{x+2}{\sqrt{(x-2)(x-3)}} dx$

SECTION – D (CASE STUDY QUESTIONS)

34. A potter made a mud vessel, where the shape of the pot is based on the function

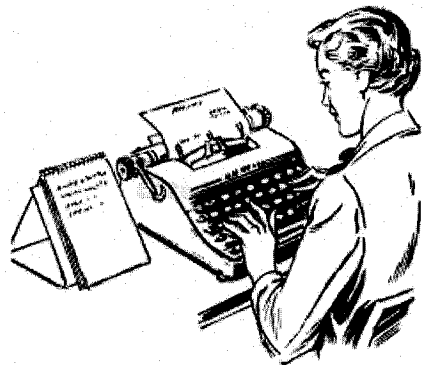
$f(x) = |x - 3| + |x - 4|$, where $f(x)$ represents the height of the pot.



Based on the given information, answer **any 4** of the following questions: (1 mark each)

- (i) when $x > 5$, what will be height of the pot in terms of x ?
(a) $2x - 7$ (b) $7 - 2x$ (c) $x - 4$ (d) $x - 3$
- (ii) when $3 < x < 4$, then the function is
(a) $2x - 7$ (b) $7 - 2x$ (c) 1 (d) -7
- (iii) Find $f'(4)$
(a) 1 (b) Function is not differentiable (c) -7 (d) -1
- (iv) The point(s) of discontinuity of $f(x)$ are _____.
(a) $x = 3$ (b) $x = 4$ (c) $x = 3$ and $x = 4$ (d) No points of discontinuity
- (v) If the potter tries to make a pot using the function $f(x) = [x]$, will he get a pot or not? why?
(a) Yes, because it is a continuous function.
(b) Yes, because it is not a continuous function.
(c) No, because it is a continuous function.
(d) No, because it is not a continuous function.

35. A professional typist Raj charges ₹ 1450 for typing 20 English, 17 Hindi and 3 Arabic pages, while Ketan charges ₹ 1800 for typing 14 English, 20 Hindi and 4 Arabic pages and Suraj charges ₹ 1650 for typing 17 English, 18 Hindi and 3 Arabic pages.



Based on the given information, answer the following questions: (2 marks each)

- (i) Assuming that the charges (in ₹) for typing one English page be x , that of one Hindi page be y and one Arabic page be z , represent the above given situation using matrices.
- (ii) If the matrix A can be expressed as the sum of a symmetric matrix P and a skew-symmetric matrix Q , find matrix P .

SECTION – E (Questions 36 to 38 carry 6 mark each)

36. If $A = \begin{bmatrix} 3 & 1 & 2 \\ 3 & 2 & -3 \\ 2 & 0 & -1 \end{bmatrix}$, find A^{-1} . Hence, solve the system of equations:

$$\begin{aligned} 3x + 3y + 2z &= 1 \\ x + 2y &= 4 \\ 2x - 3y - z &= 5 \end{aligned}$$

(OR)

Given that $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, compute AB . Use this product to solve the system of equations: $x - y = 3$; $2x + 3y + 4z = 17$; $y + 2z = 7$

37. Let $A = \{1, 2, 3, \dots, 9\}$. Show that the relation R in the set $A \times A$ defined by $(a, b)R(c, d)$ if $a + d = b + c$ for all $a, b, c, d \in A$, is an equivalence relation. Also find the equivalence class $[(2, 5)]$.
38. A rectangle is inscribed in a semicircle of radius r with one of its sides on the diameter of the semicircle. Find the dimensions of the rectangle, so that its area is maximum. Also find the maximum area.

(OR)

Prove that the height of the cylinder of maximum volume, that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

******END OF THE QUESTION PAPER******

ROLL NUMBER				
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SET	B
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- Section- E comprises of 3 questions of **6 marks** each (Q 36 to Q 38).

There is no overall choice. However, internal choice has been provided.

SECTION – A (Questions 1 to 20 carry 1 mark each)

MULTIPLE CHOICE QUESTIONS:

- If for any 2×2 square matrix A , if $A(adj A) = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$, then the value of $|A|$ is
(a) 6 (b) 36 (c) 8 (d) 1
- If $y = a \sin^3 t$, $x = a \cos^3 t$, then $\frac{dy}{dx}$ at $t = \frac{2\pi}{3}$ is
(a) -1 (b) $-\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) 1
- $\int x^2 \cos x^3 dx = ?$
(a) $-\frac{1}{3} \sin x^3 + C$ (b) $\frac{1}{3} \sin x^3 + C$ (c) $\frac{1}{3} \cos x^3 + C$ (d) $-\frac{1}{3} \cos x^3 + C$
- If A is a square matrix such that $A^2 = A$, then $(I + A)^2 - 3A$ is
(a) I (b) $2A$ (c) $3I$ (d) A
- The point on the curve $y^2 = 8x$ for which the abscissa and ordinate change at the same rate is
(a) (1, 8) (b) (0, 0) (c) (2, 4) (d) (3, -2)
- If the rate of change of area of a circle is equal to the rate of change of its radius, then its radius is
(a) $\frac{1}{2\pi}$ units (b) $\frac{1}{4\pi}$ units (c) $\frac{1}{2\sqrt{\pi}}$ units (d) $\frac{1}{\sqrt{\pi}}$ units
- The value of k so that the function $f(x) = \begin{cases} \frac{k(x-1)}{|x-1|}, & \text{if } x < 1 \\ 3, & \text{if } x \geq 1 \end{cases}$ is continuous at $x = 1$ is :
(a) 3 (b) -3 (c) 0 (d) does not exist

8. $\int \frac{\sin \sqrt{x}}{\sqrt{x}} dx$ equals
 (a) $-2 \cos \sqrt{x} + C$ (b) $\frac{1}{2} \cos \sqrt{x} + C$ (c) $2 \sin \sqrt{x} + C$ (d) $\log |\sqrt{x}| + C$
9. $\int \sqrt{1 - \sin 2x} dx =$
 (a) $\tan x - \cot x + C$ (b) $\sin x - \cos x + C$
 (c) $\sin x + \cos x + C$ (d) $\cos x - \sin x + C$
10. If $f(x) = 9^{3x-x^3}$, then $f'(-1)$ is
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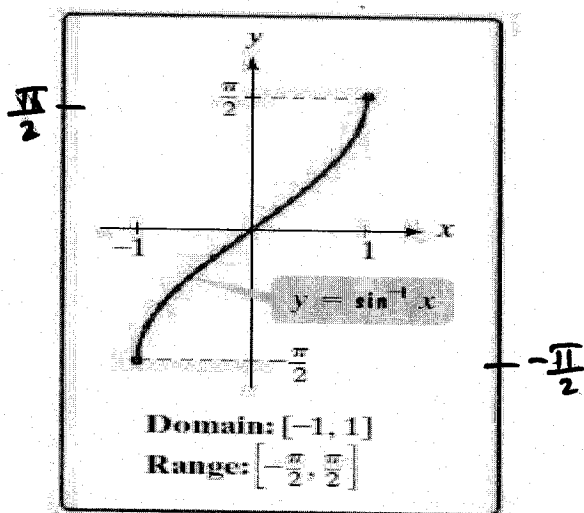
(ii) **FILL IN THE BLANKS:**

11. If $y = \cos\left(\frac{\pi}{2} - \sin^{-1} x\right)$, where $x \in [-1, 1]$, then $\frac{dy}{dx} =$ _____
12. Assume that the function $f(x)$ has its second derivative at $x = c$ such that $f'(c) = 0$ and $f''(c) > 0$, then $x = c$ is a point of _____
13. If A and B are symmetric matrices, then $(AB - BA)$ is a _____ matrix.
14. $\int e^{3 \log x} \cdot x^4 dx =$ _____

(iii) **TRUE OR FALSE :**

15. The cofactor of the element a_{21} of the determinant $\begin{vmatrix} 5 & 6 & -3 \\ -4 & 3 & 2 \\ -4 & -7 & 3 \end{vmatrix}$ is -3.
16. The total revenue in ₹ received from the sale of x units of an article is given by $R(x) = 3x^2 + 36x + 5$. The marginal revenue (in ₹) when $x = 15$ is ₹ 126.
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(iv) **ANSWER Q.19 & Q.20 BASED ON THE GIVEN GRAPH (SOURCE-BASED QUESTION):**



19. What is the value of $\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$?
20. Write one branch of $\sin^{-1} x$ other than the principal branch.

SECTION - B (Questions 21 to 25 carry 2 mark each)

21. Evaluate: $\int \frac{dx}{\sqrt{5-4x-2x^2}}$

22. Check if the relation R on the set $A = \{1, 2, 3, 4, 5, 6\}$ defined as $R = \{(x, y) : y \text{ is divisible by } x\}$ is (i) symmetric (ii) transitive.
(OR)
Show that the relation R in the set of real numbers, defined as $R = \{(x, y) : x \leq y^2\}$ is neither reflexive nor symmetric nor transitive.
23. Show that the function f in $A = R - \left\{\frac{2}{3}\right\}$ defined as $f(x) = \frac{4x+3}{6x-4}$ is one-one.
24. Differentiate: $\tan^{-1} \left(\frac{\cos x - \sin x}{\cos x + \sin x} \right)$ with respect to x .
(OR)
Find the derivative of $\sin^2 x$ with respect to $e^{\cos x}$.
25. Find $\int \frac{1}{(1+x)(2+x)} dx$
(OR)
Find $\int \frac{(x-3)e^x}{(x-1)^3} dx$

SECTION – C (Questions 26 to 33 carry 3 mark each)

26. Find the equation of the line joining $A(1, 3)$ and $B(0, 0)$ using determinants and find k if $D(k, 0)$ is a point such that area of $\triangle ABD$ is 3 sq. units.
27. Find the absolute maximum and absolute minimum values of the function given by $f(x) = \sin^2 x - \cos x$, $x \in [0, \pi]$
(OR)
Find the intervals in which the function f given by $f(x) = x^4 - 2x^2$ is
(i) strictly increasing (ii) strictly decreasing.
28. Evaluate : $\int \frac{6x+7}{\sqrt{(x-5)(x-4)}} dx$
29. If $y = (\sin^{-1} x)^2$, prove that $(1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - 2 = 0$
30. Find the value of : $\tan^{-1} \left[2 \sin \left(2 \cos^{-1} \left(-\frac{\sqrt{3}}{2} \right) \right) + 2\sqrt{3} \right]$
(OR)
Find the principal value of : $\cos^{-1} \left(\cos \frac{13\pi}{3} \right) + \tan^{-1} \left(\tan \frac{5\pi}{6} \right) - \cot^{-1}(-\sqrt{3})$
31. Evaluate : $\int x^2 \tan^{-1} x dx$.
32. If $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find x and y so that $A^2 - xI - yA = O$. Hence find A^{-1} .
(OR)
Find the value of x if $\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = O$
33. The radius of a right circular cylinder is decreasing at the rate of 3 cm/min and its height h is increasing at the rate of 2 cm/min. Find the rate of change of the volume of cylinder when $r = 7$ cm and $h = 2$ cm. (Use $\pi = \frac{22}{7}$)

SECTION – D (CASE STUDY QUESTIONS)

34. A professional typist Raj charges ₹ 1450 for typing 20 English, 17 Hindi and 3 Arabic pages, while Ketan charges ₹ 1800 for typing 14 English, 20 Hindi and 4 Arabic pages and Suraj charges ₹ 1650 for typing 17 English, 18 Hindi and 3 Arabic pages.

Based on the given information, answer the following questions: (2 marks each)

- (i) Assuming that the charges (in ₹) for typing one English page be x , that of one Hindi page be y and one Arabic page be z , represent the above given situation using matrices.
(ii) If the matrix A can be expressed as the sum of a symmetric matrix P and a skew-symmetric matrix Q , find matrix P .



35. A potter made a mud vessel, where the shape of the pot is based on the function $f(x) = |x - 3| + |x - 4|$, where $f(x)$ represents the height of the pot.



Based on the given information, answer ANY 4 of the following questions: (1 mark each)

- (i) When $x > 5$, what will be the height of the pot in terms of x ?
(a) $2x - 7$ (b) $7 - 2x$ (c) $x - 4$ (d) $x - 3$
- (ii) When $3 < x < 4$, then the function is
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- (iii) Find $f'(3)$
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(a) Yes, because it is a continuous function.
(b) Yes, because it is not a continuous function.
(c) No, because it is a continuous function.
(d) No, because it is not a continuous function.

SECTION – E (Questions 36 to 38 carry 6 mark each)

36. A rectangle is inscribed in a semicircle of radius r with one of its sides on the diameter of the semicircle. Find the dimensions of the rectangle, so that its area is maximum. Also find maximum area.

(OR)

Prove that the height of the cylinder of maximum volume, that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume

37. If $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, find A^{-1} . Hence, solve the system of equations:

$$x - 2y = 10$$

$$2x - y - z = 8$$

$$-2y + z = 7$$

(OR)

Given that $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$, and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, compute AB . Use this product to solve the system of equations: $x - y + z = 4$; $x - 2y - 2z = 9$; $2x + y + 3z = 1$

38. Let $A = \{1, 2, 3, \dots, 9\}$. Show that the relation R in the set $A \times A$ defined by $(a, b)R(c, d)$ if $a + d = b + c$ for all $a, b, c, d \in A$, is an equivalence relation. Also find the equivalence class $[(2, 5)]$.

******END OF THE QUESTION PAPER******

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 - (f) Section- E comprises of 3 questions of **6 marks** each (Q 36 to Q 38).
- There is no overall choice. However, internal choice has been provided.

SECTION – A (Questions 1 to 20 carry 1 mark each)

(i) MULTIPLE CHOICE QUESTIONS:

1. $\int x^3 \sin x^4 dx = ?$
 (a) $-\frac{1}{4} \cos x^4 + C$ (b) $\frac{1}{4} \cos x^4 + C$ (c) $\frac{1}{4} \sin x^4 + C$ (d) $-\frac{1}{4} \sin x^4 + C$
2. If the rate of change of total surface area of a sphere is equal to the rate of change of its radius, then its radius is
 (a) $\frac{1}{2\pi}$ units (b) $\frac{1}{8\pi}$ units (c) $\frac{1}{2\sqrt{\pi}}$ units (d) $\frac{1}{\sqrt{\pi}}$ units
3. If $y = a \tan^2 t$, $x = a \cot^2 t$, then $\frac{dy}{dx}$ at $t = \frac{3\pi}{4}$ is
 (a) -1 (b) $-\frac{1}{\sqrt{3}}$ (c) $-\sqrt{3}$ (d) 1
4. If A is a square matrix such that $A^2 = 2A$, then $(I + A)^2 - 4A$ is
 (a) I (b) $2A$ (c) $3I$ (d) A
5. $\int \sqrt{1 + \sin 2x} dx =$
 (a) $\tan x - \cot x + C$ (b) $\sin x - \cos x + C$
 (c) $\sin x + \cos x + C$ (d) $\cos x - \sin x + C$
6. If $f(x) = 9^{x^2+2x}$, then $f'(-1)$ is
 (a) 0 (b) 2 (c) 4 (d) -2
7. If for any 2×2 square matrix A , if $A(\text{adj } A) = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$, then the value of $|A|$ is
 (a) 49 (b) 1 (c) 7 (d) 0

8. The value of k so that the function $f(x) = \begin{cases} \frac{k(x-2)}{|x-2|}, & \text{if } x < 2 \\ 3, & \text{if } x \geq 2 \end{cases}$ is continuous at $x = 2$ is
 (a) 3 (b) -3 (c) 0 (d) does not exist
9. $\int \frac{\sin(\log x)}{x} dx$ equals
 (a) $-\cos(\log x) + C$ (b) $\frac{1}{x} \cos(\log x) + C$ (c) $\cos(\frac{1}{x}) + C$ (d) $\cos(\log x) + C$
10. The point on the curve $y^2 = 8x$ for which the abscissa and ordinate change at the same rate is:
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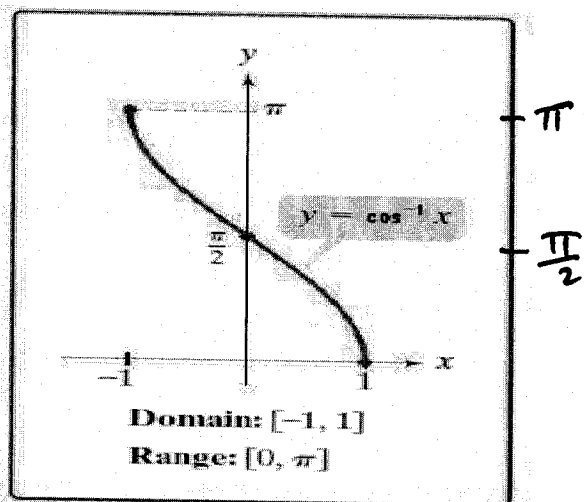
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11. If A and B are symmetric matrices, then $(AB - BA)$ is a ----- matrix.
12. If $y = \sin\left(\frac{\pi}{2} - \cos^{-1} x\right)$, where $x \in [-1, 1]$, then $\frac{dy}{dx} =$ _____
13. $\int e^{4 \log x} \cdot x^2 dx =$ _____
14. Assume that the function $f(x)$ has its second derivative at $x = c$ such that $f'(c) = 0$ and $f''(c) > 0$, then $x = c$ is a point of -----

(iii) **TRUE OR FALSE :**

15. The total revenue in ₹ received from the sale of x units of an article is given by $R(x) = 3x^2 + 36x + 5$. The marginal revenue (in ₹) when $x = 15$ is ₹ 126.
16. The cofactor of the element a_{32} of the determinant $\begin{vmatrix} 5 & 6 & -3 \\ -4 & 3 & 2 \\ -4 & -7 & 3 \end{vmatrix}$ is -2.
17. The derivative of a function is unique but a function can have infinite anti-derivatives.
18. The point at which the function $f(x) = |2x - 3|$ is not differentiable is $\frac{3}{2}$

(iv) **ANSWER Q.19 & Q.20 BASED ON THE GIVEN GRAPH (SOURCE-BASED QUESTION):**



19. Write one branch of $\cos^{-1} x$ other than the principal branch.
20. What is the value of $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$?

SECTION – B (Questions 21 to 25 carry 2 mark each)

21. Differentiate: $\tan^{-1} \left(\frac{1+\cos x}{\sin x} \right)$ with respect to x .
(OR)

Find the derivative of $x^{\sin x}$ with respect to $\sin x$.

22. Evaluate : $\int \frac{dx}{\sqrt{5-4x-2x^2}}$
23. Find $\int \frac{\sin x}{(1-\cos x)(2-\cos x)} dx$ (OR) Find $\int \frac{(x-3)e^x}{(x-1)^3} dx$
24. Show that the function f in $A = R - \left\{ \frac{2}{3} \right\}$ defined as $f(x) = \frac{4x+3}{6x-4}$ is one-one.
25. Check if the relation R on the set $A = \{1, 2, 3, 4, 5, 6\}$ defined as $R = \{(x, y): y \text{ is divisible by } x\}$ is (i) symmetric (ii) transitive.
(OR)

Show that the relation R in the set of real numbers, defined as $R = \{(a, b): a \leq b^2\}$ is neither reflexive nor symmetric nor transitive.

SECTION – C (Questions 26 to 33 carry 3 mark each)

26. If $y = (\sin^{-1} x)^2$, prove that $(1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - 2 = 0$
27. If $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find x and y so that $A^2 - xI - yA = O$. Hence find A^{-1} .
(OR)
Find the value of x if $\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$
28. The radius of a right circular cylinder is decreasing at the rate of 3 cm/min and its height h is increasing at the rate of 2 cm/min. Find the rate of change of the volume of cylinder when $r = 7$ cm and $h = 2$ cm. (Use $\pi = \frac{22}{7}$)
29. Find the absolute maximum and absolute minimum values of the function given by $f(x) = \sin^2 x - \cos x$, $x \in [0, \pi]$
(OR)
Find the intervals in which the function f given by $f(x) = x^4 - 2x^2$ is
(i) strictly increasing (ii) strictly decreasing.
30. Evaluate : $\int x^2 \cos^{-1} x \, dx$.
31. Find the equation of the line joining $A(1, 3)$ and $B(0, 0)$ using determinants and find k if $D(k, 0)$ is a point such that area of $\triangle ABD$ is 3 sq. units.
32. Find the value of : $\tan^{-1} \left[2 \sin \left(2 \cos^{-1} \left(-\frac{\sqrt{3}}{2} \right) \right) + 2\sqrt{3} \right]$
(OR)
Find the principal value of : $\tan^{-1} \left(\tan \frac{5\pi}{6} \right) - \cot^{-1}(-\sqrt{3}) + \cos^{-1} \left(\cos \frac{13\pi}{3} \right)$
33. Evaluate : $\int \frac{6x+7}{\sqrt{(x-5)(x-4)}} dx$

SECTION – D (CASE STUDY QUESTIONS)

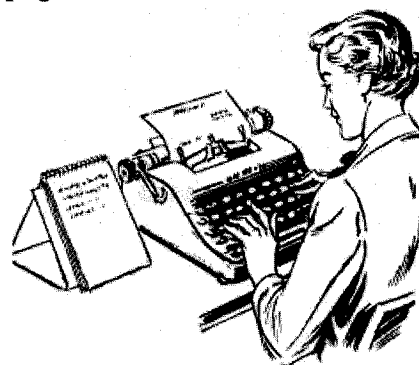
34. A potter made a mud vessel, where the shape of the pot is based on the function $f(x) = |x - 3| + |x - 4|$, where $f(x)$ represents the height of the pot.



Based on the given information, answer ANY 4 of the following questions: (1 mark each)

- (i) When $x > 5$, what will be the height of the pot in terms of x ?
(a) $2x - 7$ (b) $7 - 2x$ (c) $x - 4$ (d) $x - 3$
- (ii) When $3 < x < 4$, then the function is
(a) $2x - 7$ (b) $7 - 2x$ (c) 1 (d) -7
- (iii) Find $f'(4)$
(a) 1 (b) Function is not differentiable (c) -7 (d) -1
- (iv) Find the point(s) of discontinuity of $f(x)$
(a) $x = 3$ (b) $x = 4$ (c) $x = 3$ and $x = 4$ (d) No points of discontinuity
- (v) If the potter tries to make a pot using the function $f(x) = [x]$, will he get a pot or not? why?
(a) Yes, because it is a continuous function.
(b) Yes, because it is not a continuous function.
(c) No, because it is a continuous function.
(d) No, because it is not a continuous function.

35. A professional typist Raj charges ₹ 1450 for typing 20 English, 17 Hindi and 3 Arabic pages, while Ketan charges ₹ 1800 for typing 14 English, 20 Hindi and 4 Arabic pages and Suraj charges ₹ 1650 for typing 17 English, 18 Hindi and 3 Arabic pages.



Based on the given information, answer the following questions: (2 marks each)

- (i) Assuming that the charges (in ₹) for typing one English page be x , that of one Hindi page be y and one Arabic page be z , represent the above given situation using matrices.
- (ii) If the matrix A can be expressed as the sum of a symmetric matrix P and a skew-symmetric matrix Q , find matrix P .

SECTION – E (Questions 36 to 38 carry 6 mark each)

36. Let $A = \{1, 2, 3, \dots, 9\}$. Show that the relation R in the set $A \times A$ defined by $(a, b)R(c, d)$ if $a + d = b + c$ for all $a, b, c, d \in A$, is an equivalence relation. Also find the equivalence class $[(3, 6)]$.
37. A rectangle is inscribed in a semicircle of radius r with one of its sides on the diameter of the semicircle. Find the dimensions of the rectangle, so that its area is maximum. Also find maximum area.

(OR)

Prove that the height of the cylinder of maximum volume, that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume

38. If $A = \begin{bmatrix} 3 & 1 & 2 \\ 3 & 2 & -3 \\ 2 & 0 & -1 \end{bmatrix}$, find A^{-1} . Hence, solve the system of equations:

$$3x + 3y + 2z = 1$$

$$x + 2y = 4$$

$$2x - 3y - z = 5$$

(OR)

Given that $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, compute AB . Use this product to solve the system of equations : $x - y = 3$; $2x + 3y + 4z = 17$; $y + 2z = 7$

******END OF THE QUESTION PAPER******

